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On Bargaining Sets of Convex NTU Games

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We show that the Aumann–Davis–Maschler bargaining set and the Mas-Colell bargaining set of a non-leveled NTU game that is either ordinal convex or coalition merge convex coincides with the core of the game. Moreover, we show by means of an example that the foregoing statement may not be valid if the NTU game is marginal convex.

Keywords: NTU game; convex game; bargaining set.

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1. Introduction

Convex TU games were introduced by Shapley [1971] who discussed their basic properties and applications. One distinguished property of the family of convex games is that many of the leading solutions of TU games coincide on it. For example, Shapley has already proved in his aforementioned paper that the (non-empty) core of a convex game coincides with its (unique) von Neumann Morgenstern solution. Clearly, this result makes the core look more intuitive. Also Shapley showed, in the same paper, that the Shapley value of a convex game is a member of its core, which makes the value look more intuitive.

A second step was taken by Maschler *et al.* [1972] who proved that the kernel of a convex TU game coincides with its nucleolus and the core coincides with its (Aumann–Davis–Maschler) bargaining set. Clearly these results enforce the intuitive meaning of both the core and the bargaining set. (Indeed, Maschler [1976] claims that for some games the Aumann–Davis–Maschler bargaining set has an advantage over the core.) This paper is the starting point of our investigation: We inquire whether the core and various bargaining sets coincide for convex NTU games.

Ordinal convexity for NTU games was introduced by Vilkov [1977] who generalized some of Shapley's [1971] results (under restrictive conditions). Peleg [1986] proved that the core of an ordinal convex NTU game coincides with the von Neumann Morgenstern solution. In this paper, we investigate the bargaining set and the Mas-Colell bargaining set of ordinal convex and coalition merge convex NTU games (see Secs. 2 and 4 for the terminology), and prove their coincidence with the core (under the assumption of non-levelness).

2. Preliminaries

Let N be a finite non-empty set. For $S \subseteq N$, we denote by $\mathbb{R}^S$ the set of all real functions on S. If $x, y \in \mathbb{R}^S$, then we write $x \geq y$ if $x_i \geq y_i$ for all $i \in S$. Moreover, we write $x > y$ if $x \geq y$ and $x \neq y$ and we write $x \gg y$ if $x_i > y_i$ for all $i \in S$. Denote $\mathbb{R}^S_+ = \{x \in \mathbb{R}^S \,|\, x \geq 0\}$. A set $C \subseteq \mathbb{R}^S$ is comprehensive if $x \in C$, $y \in \mathbb{R}^S$, and $y \leq x$ imply that $y \in C$. An *NTU game* with the player set N is a pair (N, V) where V is a function which associates with every *coalition* S (that is, $S \subseteq N$ and $S \neq \emptyset$) a set $V(S) \subsetneq \mathbb{R}^S$, $V(S) \neq \emptyset$, such that

(1) $V(S)$ is closed and comprehensive;
(2) $V(S) \cap (x_S + \mathbb{R}^S_+)$ is bounded, where $x \in \mathbb{R}^N$ is defined by $x_i = \max V(\{i\})$ for $i \in N$.

Moreover, we assume that $V(\emptyset) = \emptyset$.

Let (N, V) be an NTU game. Abbreviating "boundary" by "∂" we have

$$\partial V(N) = \{x \in V(N) \,|\, \text{there exists no } y \in V(N) \text{ such that } y \gg x\},$$

i.e., $\partial V(N)$ is the set of weakly Pareto optimal elements of $V(N)$. Note that for any $\emptyset \neq S \subseteq N$, $x \in \mathbb{R}^S$ is Pareto optimal in $V(S)$ if $x \in V(S)$ and if $y \in V(S)$ and $y \geq x$ imply $x = y$. Note that, if (N, V) is *non-leveled*, i.e., for all $\emptyset \neq S \subseteq N$ and all $x, y \in \partial V(S)$, $x \geq y$ implies $x = y$, then $\partial V(S)$ is the set of Pareto optimal elements in $V(S)$.

In order to recall the definitions of the unconstrained (Aumann–Davis–Maschler) bargaining set [Aumann and Maschler, 1964; Davis and Maschler, 1967] and of the Mas-Colell prebargaining set [Mas-Colell, 1989], let $x \in \mathbb{R}^N$. A pair (P, y) is an *objection* at x (of any player in P against any player in $N \backslash P$) if $\emptyset \neq P \subseteq N$, $y \in V(P)$, and $y > x_P$. An objection (P, y) is *strong* if $y \gg x_P$. The pair (Q, z) is a *weak counter objection* to the objection (P, y) if $\emptyset \neq Q \subseteq N$, $z \in V(Q)$, and $z \geq (y_{P \cap Q}, x_{Q \backslash P})$. A weak counter objection (Q, z) is a *counter objection* to the objection (P, y) if $z > (y_{P \cap Q}, x_{Q \backslash P})$. A strong objection (P, y) is *justified in the sense of the bargaining set* if there exist players $k \in P$ and $\ell \in N \backslash P$ such that there does not exist any weak counter objection (Q, z) to (P, y) satisfying $\ell \in Q$ and $k \notin Q$. The *unconstrained bargaining set* of (N, V), $\mathcal{PM}(N, V)$, is the set of all $x \in \partial V(N)$ that do not have strong justified objections at x in the sense of the bargaining set [Davis and Maschler, 1967]. An objection (P, y) is *justified in the sense of the Mas-Colell bargaining set* if there does not exist any counter objection to (P, y). The *Mas-Colell prebargaining set* of (N, V), $\mathcal{PMB}(N, V)$, is the set of all $x \in \partial V(N)$ that do not have a justified objection at x in the sense of the Mas-Colell bargaining set [Mas-Colell, 1989].

Note that the *bargaining set*, $\mathcal{M}(N, V)$, is defined by $\mathcal{M}(N, V) = \mathcal{PM}(N, V) \cap I(N, V)$ and the *Mas-Colell bargaining set*, $\mathcal{MB}(N, V)$, is defined by $\mathcal{MB}(N, V) = \mathcal{PMB}(N, V) \cap I(N, V)$, where $I(N, V) = \{x \in \partial V(N) \,|\, x_i \geq \max V(\{i\})$ for all $i \in N\}$, i.e., $I(N, V)$ is the set of *imputations*.

Recall that (N, V) is

- *superadditive* if $V(S) \times V(T) \subseteq V(S \cup T)$ for all $S \subseteq N$ and $T \subseteq N \backslash S$;
- *ordinal convex* if for all $S, T \subseteq N$ and $x \in \mathbb{R}^N$, $x_S \in V(S)$ and $x_T \in V(T)$ imply that $x_{S \cap T} \in V(S \cap T)$ or $x_{S \cup T} \in V(S \cup T)$.

Note that an ordinal convex game is, hence, superadditive.

3. The Excess NTU Game

For an NTU game (N, V) and $x \in \mathbb{R}^N$ we define the *excess game* (N, V^x) by the requirement that, for any $\emptyset \neq S \subseteq N$,

$$V^x(S) = (-\mathbb{R}^S_+) \cup \bigcup_{\emptyset \neq T \subseteq S} (V(T) - x_T) \times (-\mathbb{R}^{S \backslash T}_+).$$

Note that with $V^x(\emptyset) = \emptyset$ the pair (N, V^x) is an NTU game (i.e., (1) and (2) are valid). Moreover, let $\mathcal{C}(N, V)$ denote the *core* of (N, V), i.e.,

$$\mathcal{C}(N, V) = \{x \in V(N) \,|\, x_S \in \mathbb{R}^S \backslash (V(S) \backslash \partial V(S)) \text{ for all } \emptyset \neq S \subseteq N\}.$$

Remark 3.1. For a non-leveled NTU game (N, V), $\mathcal{C}(N, V) \subseteq \mathcal{MB}(N, V) \cap \mathcal{M}(N, V)$ because there does not exist an objection at any element of the core.

Lemma 3.2. *Let (N, V) be an NTU game and $x \in V(N)$. Then $x \in \mathcal{C}(N, V)$ if and only if $0 \in \mathcal{C}(N, V^x)$.*

Proof. If $x \in \mathcal{C}(N, V)$, then $0 = x - x \in V^x(N)$. Moreover, if $y \in V^x(S), y \not\leq 0 \in \mathbb{R}^S$, for some $\emptyset \neq S \subseteq N$, then there exists $\emptyset \neq R \subseteq S$ such that $y_R + x_R \in V(R)$ and $y_{S \backslash R} \leq 0$. Hence, there exists $i \in R$ such that $x_i \geq y_i + x_i$ and we conclude that $0 \in \mathcal{C}(N, V^x)$.

Conversely, if $0 \in \mathcal{C}(N, V^x)$, then, for any coalition T and any $y \in V(T)$, $y \geq x_T$ implies $y - x_T \in V^x(T)$ so that there exists $j \in T$ with $y_j - x_j \leq 0$. Thus, $x \in \mathcal{C}(N, V)$. □

We may now prove the main result of this section.

Theorem 3.3. *If (N, V) is a non-leveled game and $x \in \mathcal{PMB}(N, V)$, then $\mathcal{C}(N, V^x) \neq \emptyset$ if and only if $x \in \mathcal{C}(N, V)$.*

Proof. The "if" direction is implied by the "only if" direction of Lemma 3.2. For the remaining direction, let $\bar{x} \in \mathcal{C}(N, V^x)$ and assume, on the contrary, that $x \notin \mathcal{C}(N, V)$. Let $P = \{i \in N \mid \bar{x}_i > 0\}$. As $V^x(\{i\}) \supseteq -\mathbb{R}_+$ for all $i \in N$, $\bar{x} \geq 0$ and by the "if" direction of Lemma 3.2, $P \neq \emptyset$. As $\bar{x} \in V^x(N)$, there exists $P \subseteq S \subseteq N$ such that $\bar{x}_S + x_S \in V(S)$. Hence, $(S, \bar{x}_S + x_S)$ is an objection to x in the sense of the Mas-Colell bargaining set. Let (Q, y) be a counterobjection to $(S, \bar{x}_S + x_S)$. Then $y > (\bar{x}_{S\cap Q} + x_{S\cap Q}, x_{Q\setminus S})$. By the non-levelness of $V(Q)$ there exists $y' \in V(Q)$ such that $y' \gg (\bar{x}_{S\cap Q} + x_{S\cap Q}, x_{Q\setminus S})$. As $y' - x_Q \in V^x(Q)$ and $\bar{x}_{Q\setminus S} = 0$, $y' - x_Q \gg \bar{x}_Q$ which is impossible because $\bar{x} \in \mathcal{C}(N, V^x)$. □

The following corollary may be regarded as a generalization of Solymosi's [1999] main result for TU games.

Corollary 3.4. *If (N, V) is a superadditive non-leveled NTU game and $x \in \mathcal{PM}(N, V)$, then $\mathcal{C}(N, V^x) \neq \emptyset$ if and only if $x \in \mathcal{C}(N, V)$.*

Proof. We may assume that (N, V) is zero-normalized because the set of superadditive non-leveled NTU games on N is closed under translations and the core and the unconstrained bargaining set are translation covariant. Then (N, V) satisfies all assumptions of Holzman's [2001] Theorem 3.1 stating that $\mathcal{M}(N, V) \subseteq \mathcal{PMB}(N, V)$. His proof, however, does not use individual rationality so that, in fact, $\mathcal{PM}(N, V) \subseteq \mathcal{PMB}(N, V)$, and Theorem 3.3 finishes the proof. □

4. Results and Examples

In order to apply the results of Sec. 3 to ordinal convex NTU games, the following lemma is needed.

Lemma 4.1. *If (N, V) is an ordinal convex NTU game, then (N, V^x) is ordinal convex.*

Proof. Let $\emptyset \neq S, T \subseteq N$ and let $y \in \mathbb{R}^N$ satisfy $y_S \in V^x(S)$ and $y_T \in V^x(T)$. We have to show that $y_{S\cap T} \in V^x(S \cap T)$ or $y_{S\cup T} \in V^x(S \cup T)$. If $y_S \leq 0 \in \mathbb{R}^S$ or $y_T \leq 0 \in \mathbb{R}^T$, then $y_{S\cup T} \in V^x(S \cup T)$. Hence, we may assume that neither $y_S \leq 0$ nor $y_T \leq 0$. Then there exist $Q \subseteq S$ and $R \subseteq T$ such that $Q \neq \emptyset \neq R$, $y_Q \in V(Q) - x_Q$, $y_{S\setminus Q} \leq 0$, $y_R \in V(R) - x_R$, and $y_{T\setminus R} \leq 0$. Therefore, there exists $z \in \mathbb{R}^N$ such that $y \leq z - x$, $z_Q \in V(Q), z_R \in V(R)$, $z_i = x_i$ for all $i \in (S \cup T)\setminus(Q \cup R)$. By ordinal convexity of V, $z_{Q\cap R} \in V(Q \cap R)$ or $z_{Q\cup R} \in V(Q \cup R)$. If $z_{Q\cap R} \in V(Q \cap R)$, then $z_{S\cap T} - x_{S\cap T} \in V^x(S \cap T)$ so that $y_{S\cap T} \in V^x(S \cap T)$ by comprehensiveness. Similarly, $z_{Q\cup R} \in V(Q \cup R)$ implies that $y_{S\cup T} \in V^x(S \cup T)$. □

The core of an ordinal convex game is non-empty [Greenberg, 1985]. Moreover, an ordinal convex game is superadditive. Thus, Remark 3.1, Theorem 3.3, Lemma 3.2, and Corollary 3.4 have the following consequence.

Corollary 4.2. *The unconstrained bargaining set and the Mas-Colell prebargaining set of any ordinal convex non-leveled NTU game coincide with its core.*

The following example shows that "non-levelness" is needed in the statement concerning the unconstrained bargaining set of Corollary 4.2. Let $|N| \geq 3$ and, for any $S \subseteq N$,

$$V(S) = \begin{cases} \emptyset, & \text{if } S = \emptyset, \\ -\chi_S^S - \mathbb{R}_+^S, & \text{if } 1 \leq |S| \leq |N| - 2, \\ -\mathbb{R}_+^S, & \text{if } |S| \geq |N| - 1, \end{cases}$$

where $\chi^S \in \mathbb{R}^N$ is the characteristic vector of S, i.e., $\chi_i^S = 1$ for $i \in S$ and $\chi_j^S = 0$ for $j \in N \backslash S$. Then (N, V) is ordinal convex. Let $k \in N$ and $x = -\chi_{N\backslash\{k\}}$. Then $x \notin \mathcal{C}(N, V)$. Note that k has no objection against any other player and any objection of any $i \in N\backslash\{k\}$ is of the form (S, y) such that $S = N\backslash\{k\}$, $0 \geq y \gg x_{N\backslash\{k\}}$, so that $(N\backslash\{i\}, 0)$ is a counterobjection. As x is individually rational, $x \in \mathcal{M}(N, V)$.

By means of an example that is derived from the voting game of the Voting Paradox [Holzman *et al.*, 2007, Sec. 3], we now show that non-levelness is also crucial for the statement concerning the Mas-Colell bargaining set.

Example 4.3. Let $N = \{1, 2, 3\}$ and (N, V) the 0-normalized game defined by

$$V(\{1,2\}) = \{(2,1), (0,2)\} - \mathbb{R}_+^{\{1,2\}},$$

$$V(\{1,3\}) = \{(2,0), (1,2)\} - \mathbb{R}_+^{\{1,3\}},$$

$$V(\{2,3\}) = \{(2,1), (0,2)\} - \mathbb{R}_+^{\{2,3\}}, \quad \text{and}$$

$$V(N) = \{(2,2,0), (2,0,2), (0,2,2), (2,1,1), (1,2,1), (1,1,2)\} - \mathbb{R}_+^N.$$

For any $\emptyset \neq S, T \subseteq N$ and any $x \in \mathbb{R}^N$ such that $x_S \in V(S)$ and $x_T \in V(T)$ we have $x_{S\cup T} \in V(S \cup T)$. Indeed, in order to verify this fact we may assume that $S\backslash T \neq \emptyset \neq T\backslash S$. If $|S| = |T| = 1$, then $x_{S\cup T} \leq 0 \in V(S \cup T)$, otherwise, $S \cup T = N$. If $x \not\gg 0$, say $x_i \leq 0$, then $x_j \leq 2$ for all $j \in N$ implies $x \in V(N)$. Finally, if $x \gg 0$, then $|S| = |T| = 2$ and $x \leq (2,1,1)$ or $x \leq (1,2,1)$ or $x \leq (1,1,2)$. Hence, (N, V) is ordinal convex.

Let $y = (1, 1, 0)$. Then y is weakly Pareto optimal. Assume that y has a justified weak objection (P, z). Then z is Pareto optimal in $V(P)$. If $P = N$ and $z = (2,2,0)$, $z = (2,0,2)$, or $z = (0,2,2)$, then (P, z) can be countered by $(\{2,3\}, (2,1))$, $(\{1,2\}, (2,1))$, or $(\{1,3\}, (1,2))$, respectively. If $z = (2,1,1), z = (1,1,2)$, or $z = (1,2,1)$, then (P, z) can also be countered by the aforementioned pairs, respectively. If $P = \{1,2\}$, then $z = (2,1)$ so that $(\{2,3\}, (2,1))$ is a counterobjection. If $P = \{2,3\}$, then $z = (2,1)$ so that $(\{1,3\}, (1,2))$ is a counterobjection. Finally, if $P = \{1,3\}$, then either $z = (2,0)$ so that $(\{2,3\}, (2,1))$ is a counterobjection or $z = (1,2)$ so that $(\{1,2\}, (2,1))$ is a counterobjection. Hence $y \in \mathcal{MB}(N, V)$.

Moreover, $(2,1) \in V(\{2,3\})$. Thus $\mathcal{MB}(N,V)\setminus\mathcal{C}(N,V) \neq \emptyset$. Also, $(2,2,0) \in \mathcal{C}(N,V)$ has the justified objection $(\{2,3\},(2,1))$ so that $\mathcal{C}(N,V)\setminus\mathcal{MB}(N,V) \neq \emptyset$.

For $|N| = 2$, the bargaining set $\mathcal{M}$ coincides with and the Mas-Colell prebargaining set $\mathcal{PMB}$ is contained in the core, provided that the core is non-empty. If (N,V) is defined by $V(S) = -\mathbb{R}^S_+$ for all $S \subseteq N$, then $\mathcal{C}(N,V) = \{0\}$ and 0 is the unique individually rational feasible payoff vector so that the bargaining sets coincide with the core. However, any $x \in \mathbb{R}^N$ satisfying $x \leq 0$, but $x \not\ll 0$ (i.e., $x_i = 0$ for some $i \in N$) belongs to $\mathcal{PM}(N,V)$. However, $\mathcal{PMB}(N,V) = \mathcal{C}(N,V)$.

For any finite non-empty set N let $\Pi(N)$ denote the set of *orderings* of N, i.e.,

$$\Pi(N) = \{\pi : N \to \{1,\ldots,|N|\} \mid \pi \text{ is bijective}\}.$$

Moreover, for $i \in N$ and $\pi \in \Pi(N)$, denote $P_i^\pi = \{j \in N \mid \pi(j) < \pi(i)\}$ and define, recursively, $x_i^{V,\pi} \in \mathbb{R} \cup \{-\infty\}$, $i = \pi^{-1}(1),\ldots,\pi^{-1}(|N|)$, by

$$x_i^{V,\pi} = \begin{cases} 0, & \text{if there exists } j \in P_i^\pi \text{ with } x_j^{V,\pi} = -\infty, \\ \sup\{x_i \in \mathbb{R} \mid (x_i, x^{V,\pi}_{P_i^\pi}) \in V(P_i^\pi \cup \{i\})\}, & \text{otherwise,} \end{cases}$$

where $\sup\emptyset = -\infty$. The game is called *marginal convex* if, for all $\pi \in \Pi(N)$, $x^{V,\pi} \in \mathcal{C}(N,V)$.

Example 4.4. Let (N,V) be the 0-normalized game defined by Asscher [1976, Example 4.1], that is, $N = \{1,2,3\}$, for any $S \subseteq N$ with $|S| = 2$, $x \in V(S)$ if and only if $x_k + x_\ell \leq 210$ and $x_k + 3x_\ell \leq 450$ for all $k \in S$, where $S\setminus\{k\} = \{\ell\}$, and $V(N) = \{x \in \mathbb{R}^N \mid x_1 + x_2 + x_3 \leq 300\}$. Then $x = (100,100,100) \notin \mathcal{C}(N,V)$, but, by a simple symmetry argument, $x \in \mathcal{MB}(N,V) \cap \mathcal{M}(N,V)$. Moreover, $\{x^{V,\pi} \mid \pi \in \Pi(N)\} = \{(0,150,150),(150,0,150),(150,150,0)\} = \mathcal{C}(N,V)$ so that (N,V) is a non-leveled convex-valued marginal convex game.

A game (N,V) is *coalition merge convex* if (N,V) is superadditive and if the following condition is satisfied: For any $\emptyset \neq R,S,T \subseteq N$ with $S \subsetneq T \subseteq N\setminus R$, any $x \in \partial V(S)$ satisfying $x_i \geq \max V(\{i\})$ for all $i \in S$, any $y \in V(T)$, and any $z \in \mathbb{R}^R$ such that $(x,z) \in V(S \cup R)$, $(y,z) \in V(T \cup R)$.

According to Csóka *et al.* [2011], a coalition merge convex game is marginal convex.

Remark 4.5. If (N,V) is coalition merge convex and $x \in \mathbb{R}^N$, then (N,V^x) is coalition merge convex. Hence our corollary may be extended: The unconstrained bargaining set and the Mas-Colell prebargaining set of any coalition merge convex non-leveled NTU game coincide with its core.

Remark 4.6. The core of a *cardinal convex* game (N,V) is non-empty provided that $V(N)$ is a convex set [Sharkey, 1981]. It can easily be verified that the excess game (N,V^x) of a cardinal convex game (N,V) is itself cardinal convex. However, even if $V(S)$ is convex for each $S \subseteq N$, then, as a union of convex sets, $V^x(N)$ may not be convex. We do not know if Corollary 4.2 holds for cardinal convex games with convex $V(N)$.

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