

Professor Bezalel Peleg (1936–2019)

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With Bezalel Peleg's death on May 9, 2019, the social choice and game theory communities have lost a pioneering and influential member. Bezalel Peleg completed his Ph.D. in mathematics at the Hebrew University of Jerusalem under the supervision of Robert Aumann in 1964. His first paper (Peleg 1959), on solvable n -person games, appeared in the *Bulletin of the American Mathematical Society*. He was a productive researcher during the last sixty years in which he was affiliated with the Hebrew University of Jerusalem. His scientific work covers many areas in economic theory, including cooperative and non-cooperative game theory, general equilibrium theory, and social choice theory. For an extensive bibliography and discussion of his work until 2012, see (Peters and Sudhölter 2012). Here, we just highlight a few topics that we believe are central in his work and were central in his mind.

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His first and seminal paper in social choice theory (Peleg 1978) paved the way for his view that implementation theory should not only deal with equilibria of the game form (or mechanism) resulting in sincere outcomes, but also with the game form itself representing the social system to be implemented. If a nation's constitution describes, for each group in society, the set of social states to which this group is entitled, then the system of laws and rules should reflect exactly this: it should not give any group more or less power than is determined by the constitution. Additionally, given the preferences of the members of society over the possible social states, the laws and rules in vigor should allow for the existence of a stable society. In technical terms, a constitution can be formalized as an effectivity function, and a system of laws and rules as a game form which exactly represents the effectivity function. This game form should have an equilibrium—in this case a strong equilibrium. If this game form is in particular a social choice function—hence, a direct mechanism having the agents' preferences as strategies—then this social choice function appears to be closely related to a special kind of veto voting, called feasible elimination, a concept that reappears in Bezael Peleg's work on social choice. Effectivity functions result from social choice functions or correspondences, game forms, simple games, and more. Bezael Peleg regarded the effectivity function (see his paper with Moulin, 1982) not just as a technical tool but rather as a central concept that is basic for describing what groups of agents are entitled to or capable of—like in the mentioned case of a nation's constitution. Thus, they occupy an important position in his work.

Already his first paper (Peleg 1959) considers cooperative games, namely those that are superadditive and possess von Neumann-Morgenstern stable sets, and Bezael Peleg contributed to cooperative game theory throughout until now. In a seminal paper with Aumann (1960) he extended stable sets to cooperative games without side payments (NTU games) and formulated the notions of α - and β -effectiveness for cooperative games in strategic form that lead to α - and β -versions of solutions like the core. In another early paper (1967) he proved the existence of the bargaining set, thereby providing a generalization of the Knaster-Kuratowski-Mazurkiewicz lemma. In an important paper with Maschler and Shapley (1967) he proved that the bargaining set and kernel of a convex game coincide with its core and nucleolus, respectively. In the same year, with Maschler, he supported several kinds of generalized nucleoli as stable points of certain intuitively defined set-valued dynamic systems. Some of his later major contributions belong to the so-called axiomatic approach. Based on the Davis-Maschler reduced game, he extended the reduced game property to set-valued solutions and introduced the converse reduced game property. With the help of these and other intuitive axioms he provided axiomatizations of the core for NTU games (1985), for market games (1989, and with Sudhölter, 2002), and of the prekernel. His popular book (Peleg and Sudhölter 2007) on cooperative game theory provides a systematic study of the core, kernel, bargaining sets, and the Shapley value. He analyzed and compared the Davis-Maschler bargaining set and the Mas-Colell bargaining set for simple majority NTU voting games (with Holzman and Sudhölter, 2007) and, more recently, showed that these bargaining sets coincide with the core for certain kinds of convex NTU games, namely for ordinal and for coalition merge convex games (with Sudhölter, 2015).

An important contribution of Bezalel Peleg to non-cooperative game theory was the development of a new solution concept (with Bernheim and Whinston, 1987) called coalition-proof Nash equilibrium. A coalition-proof Nash equilibrium is a strategy profile such that no coalition can credibly and profitably deviate. A profitable deviation is not credible if there is a subcoalition that can credibly and profitably deviate from the original deviation. This leads to a recursive definition, where profitable deviations by one-person coalitions are always credible. Coalition proof Nash equilibrium is a weakening of strong equilibrium; the latter often fails to exist.

Bezalel Peleg's attitude as a theorist was characterized by his striving for both mathematical rigor and meaningfulness. His writing style was short and precise, and he regarded the problems that he worked on as important and pertaining to real-life. He would not easily compromise or be satisfied by partial solutions. Bezalel Peleg was an extremely modest and friendly person whom we will miss dearly.

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