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ORIGINAL ARTICLE

Average monotonic cooperative games with nontransferable utility

José-Manuel Giménez-Gómez[1] · Peter Sudhölter[2] · Cori Vilella[3]

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Abstract
A non-negative transferable utility (TU) game is *average* monotonic if there exists a non-negative vector according to which the relative worth is not decreasing when enlarging the coalition. We generalize this definition to the nontransferable utility (NTU) case. It is shown that an average monotonic NTU game shares several properties with an average monotonic TU game. In particular it has a special core element and there exists a population monotonic allocation scheme. We show that an NTU bankruptcy game is average monotonic with respect to the claims vector.

Keywords Nontransferable utility · Average monotonicity · Core · Population monotonicity

JEL Classification C71

1 Introduction

Izquierdo and Rafels (2001) define *average monotonic* cooperative games with transferable utility that allow to model multilateral interactive decision problems in

✉ José-Manuel Giménez-Gómez
josemanuel.gimenez@urv.cat

Peter Sudhölter
psu@sam.sdu.dk

Cori Vilella
cori.vilella@urv.cat

1 Departament d'Economia and ECO-SOS, Universitat Rovira i Virgili, Av. Universitat 1, 43204 Reus, Spain

2 Department of Economics, University of Southern Denmark, Campusvej 55, 5230 Odense, Denmark

3 Dep. de Gestió d'Empreses and ECO-SOS, Universitat Rovira i Virgili, Av. Universitat 1, 43204 Reus, Spain

economic situations with increasing average profits in which side payments are possible. For instance, consider a group of investors such that each of them has an amount of money to invest, and a bank offering a yield that depends increasingly on the total amount of the deposited money. Then, if the investors can combine their resources and invest them in the bank, there are incentives to form a coalition, since increasing investments generate an increasing interest rate (see Izquierdo 1996, and Izquierdo and Rafels 2001, for further details.)

In this context an arbitrary coalition of decision makers may form and select a feasible alternative that creates a profit for each of its members. The mentioned authors now assume that the arising aggregate profit may be redistributed in an arbitrary way to its members, i.e., it is assumed that side payments are possible. Therefore, the game that suitably models such a decision problem is a cooperative *transferable utility* game, a TU game.

If, on the other hand, side payments are not possible (they may be prohibited or physically impossible), then such a situation may be modeled as a cooperative *nontransferable utility* game, an NTU game. It should be noted that a TU game may be regarded as a special NTU game.

Based on the definition of average monotonic cooperative TU games given by Izquierdo and Rafels (2001), we define average monotonic cooperative NTU games. Specifically, we generalize the definition of average monotonicity to the NTU case, showing that a TU game is average monotonic if and only if its corresponding NTU game is average monotonic. We show that an average monotonic NTU game has some properties in common with an average monotonic TU game. In fact, it turns out that, as for an average monotonic TU game, the "proportional distribution" is a remarkable core element of an average monotonic NTU game as well.

Furthermore, we show that the allocation scheme that assigns to each coalition its proportional distribution does not decrease the payoffs of the players of a coalition when they form a larger coalition. That is, the extension to all coalitions of the proportional distribution is a *population monotonic allocation scheme* in the sense of Moulin (1990) and Sprumont (1990).

Finally, we prove that every NTU *bankruptcy game* in the sense of Orshan et al. (2003) is average monotonic with respect to the claims vector.

The paper is organized as follows. In Sect. 2 we formally present some basics about TU and NTU games. In Sect. 3 we present the definition of an average monotonic NTU game with respect to (w.r.t.) a vector α and show that each subgame of such a game is average monotonic w.r.t. the restricted vector of α and that the corresponding proportional distribution is in the core of the game. In Sect. 4 we show that the extension of the proportional distribution to all coalitions is a population monotonic allocation schemes. Finally, Sect. 5 is devoted to show that NTU bankruptcy games are average monotonic w.r.t. the claims vectors.

2 Notation and basic definitions

This section is devoted to introduce notation and recall basic definitions.

2.1 General notation for vectors and sets

We start with some notations. Throughout, let N be a finite nonempty set of elements called *players*. A *coalition* S is a nonempty subset of N and we denote by s the number of players in S. By $\mathcal{N}$ we denote the set of all coalitions in N, i.e., $\mathcal{N} = 2^N \setminus \{\emptyset\}$. The elements of $\mathbb{R}^N$ will be identified with n-dimensional vectors whose coordinates are indexed by the members of N. For each coalition S denote by 0_S the zero vector of $\mathbb{R}^S$. Further, if $x \in \mathbb{R}^N$, and $S \in \mathcal{N}$ is a coalition, we write x_S for the restriction of x to S, i.e. $x_S := (x_i)_{i \in S} \in \mathbb{R}^S$ and $x(S) = \sum_{i \in S} x_i$. Moreover, for $S \in \mathcal{N}$ and $x, y \in \mathbb{R}^S$, we write $x \geq y$ if $x_i \geq y_i$ for each $i \in S$.

We say that the set $X \subseteq \mathbb{R}^S$, where $S \in \mathcal{N}$, is *comprehensive* if $x \in X$, $y \in \mathbb{R}^S$, and $x \geq y$ imply $y \in X$. Moreover, let $\mathbb{R}^S_+ := \{x \in \mathbb{R}^S | x \geq 0_S\}$ and let ∂X denote the boundary of X.

2.2 TU games and NTU games

A *TU game* (on N) is a pair (N, v) such that $v : 2^N \to \mathbb{R}$, the *coalition function* (also called *characteristic function*), satisfies $v(\emptyset) = 0$.

Using this notation we now recall a standard definition of a cooperative game without transferable utility, an NTU game.

Definition 2.1 An *NTU game* (on N) is a pair (N, V) such that V is a mapping that assigns to each coalition $S \in \mathcal{N}$ a subset $V(S)$ of $\mathbb{R}^S$ of *attainable payoff vectors* satisfying the following conditions:

(i) $V(S)$ is nonempty, closed and comprehensive,
(ii) $V(S) \cap (x_S + \mathbb{R}^S_+)$ is bounded for every $x_S \in \mathbb{R}^S$.

It is also assumed that $V(\emptyset) = \emptyset$.

It is noteworthy, in general, we do not require an NTU game (N, V) to be convex-valued, i.e., for a coalition S, $V(S)$ is not required to be convex, unless explicitly stated.

A TU game (N, v) can be considered as an NTU game in the following natural way. Indeed, let (N, V_v) be the NTU game defined by $V_v(S) = \{x \in \mathbb{R}^S \mid x(S) \leq v(S)\}$ for all $S \in \mathcal{N}$. Then we say that (N, V_v) is the NTU game *corresponding to* the TU game (N, v).

Let (N, V) be an NTU game. For every $i \in N$ let

$$v_i = \max\{x_i \mid x_{\{i\}} \in V(\{i\})\}.$$

We often identify (N, V) with its characteristic function V. The intended interpretation is that $x \in V(S)$ if cooperation within the coalition S allows to create the utility allocation x for the members of S. In order to simplify the notation we will write $V(i)$ instead of $V(\{i\})$.

For each $S \in \mathcal{N}$, by slightly abusing notation, we denote by (S, V_S) its *subgame* on S. That is, the set of players is S and $V_S(T) = V(T)$ for any $T \subseteq S$. A similar notation is used for TU games (N, v).

2.3 The core and properties of games

Let (N, V) be an NTU game. The *core* of (N, V), $\mathcal{C}(N, V)$, is the set of all vectors $x \in V(N)$ such that, for each coalition S and each allocation $y \in V(S)$, there exists $i \in S$ such that $x_i \geq y_i$. Note that core of a TU game (N, v) coincides with the core of its corresponding NTU game (N, V_v).

Next, we recall some properties of NTU games that we use. An NTU game (N, V) is *superadditive* if, for all $S, T \in \mathcal{N}$ such that $S \cap T = \emptyset$, $V(S) \times V(T) \subseteq V(S \cup T)$. Moreover, (N, V) is *weakly* superadditive if the foregoing condition is just requested in the case that $T = \{i\}$ is a singleton. Note that a TU game satisfies (weak) superadditivity if and only if its corresponding NTU game does.

We now relax weak superadditivity further and say that an NTU game (N, V) is *weakly* superadditive* if $\bigtimes_{i \in S} V(i) \subseteq V(S)$ for all $S \in \mathcal{N}$. Hence, subgames of (weakly*) superadditive games are (weakly*) superadditive.

An NTU game is *monotonic* (Hart and Mas-Colell 1996) if for all coalitions $S, T \in \mathcal{N}$ with $S \subseteq T$ and all $x \in V(S)$, there exists $y \in V(T)$ with $y_S \geq x$ and $y_{T \setminus S} \geq 0$, i.e., $V(S) \times \{0_{T \setminus S}\} \subseteq V(T)$. Note that the foregoing definition of monotonicity expands the classical definition of monotonicity for TU games, i.e., a TU game is monotonic if and only if its corresponding NTU game is.

According to Otten et al. (1998) an NTU game (N, V) is *weakly monotonic* if for all coalitions $S, T \in \mathcal{N}$ with $S \subseteq T$ and all $x \in V(S)$, there exists an $y \in V(T)$ with $y_S \geq x$, i.e., $V(S) \subseteq \{y_S \mid y \in V(T)\}$.

Note that NTU games corresponding to TU games are weakly monotonic. Monotonicity implies weak monotonicity, but there are weakly monotonic NTU games that are not monotonic

3 Average monotonic games with nontransferable utility

Izquierdo and Rafels (2001) introduce and study average monotonic TU games. We now recall the corresponding definition. The TU game (N, v) is *average monotonic* w.r.t. $\alpha \in \mathbb{R}^N_+ \backslash \{0_N\}$ if $v(S) \geq 0$ for all $S \in \mathcal{N}$ and v *does not decrease in average w.r.t.* α, *i.e., for all* $S, T \in \mathcal{N}$ *with* $S \subseteq T$, $\alpha(T)v(S) \leq \alpha(S)v(T)$. Say that the TU game (N, v) is *average monotonic* if there exists $\alpha \in \mathbb{R}^N_+ \setminus \{0_N\}$ such that (N, v) is average monotonic w.r.t. α.

Remark 3.1 Let $\alpha \in \mathbb{R}^N_+ \setminus \{0_N\}$, let (N, v) be a TU game, and let $S \in \mathcal{N}$.

(1) If (N, v) is average monotonic w.r.t. α and $x_S \neq 0_S$, then (S, v_S) is average monotonic w.r.t. α_S.

(2) If (N, v) is average monotonic w.r.t. α and $\alpha_S = 0_S$, then $\alpha(N)v(T) \leq \alpha(T)v(N) = 0$ implies $v(T) = 0$ for all $T \subseteq S$ because $\alpha(N) > 0$. However, by definition, if the subgame on S, (S, v_S), is the zero game, then it is average monotonic w.r.t. every $\alpha' \in \mathbb{R}^S_+ \backslash \{0_S\}$.

(3) Therefore, (N, v) is average monotonic if and only if $v(S) \geq 0$ for all $S \in \mathcal{N}$ and there exists $\alpha' \in \mathbb{R}^N_+$, where we do not exclude $\alpha' = 0_N$, such that, for all $S, T \in \mathcal{N}$ with $S \subseteq T$, $\alpha'(T)v(S) \leq \alpha'(S)v(T)$ and $v(S) = 0$ if $\alpha'_S = 0_S$.

From now on, for convenience, we say that the zero TU game (N, v) is average monotonic w.r.t. every $\alpha \in \mathbb{R}^N_+$, including $\alpha = 0_N$. Using this convention, we obtain the following characterization of average monotonic TU games.

Proposition 3.2 *The TU game (N, v) is average monotonic w.r.t. $\alpha \in \mathbb{R}^N_+$ if and only if the following three conditions hold:*

$$S \in \mathcal{N} \Rightarrow v(S) \geq 0 \tag{1}$$

$$S \in \mathcal{N} \text{ and } \alpha_S = 0_S \Rightarrow v(S) = 0 \tag{2}$$

$$S, T \in \mathcal{N}, q > 0, S \subseteq T, \alpha_S \neq 0_S, \alpha(S)q \leq v(S) \Rightarrow \alpha(T)q \leq v(T) \tag{3}$$

The proof can be deduced from Theorem 3.1 of Izquierdo and Rafels (2001) and it is added for completeness reasons.

Proof For the only if part, suppose that (N, v) is average monotonic w.r.t. α. Remark 3.1 (3) directly implies (1) and (2). In order to show (3), let S, T, q satisfy the required conditions. Then $q \leq \frac{v(S)}{\alpha(S)}$ and, hence, $\alpha(T)q \leq \frac{\alpha(T)v(S)}{\alpha(S)} \leq \frac{\alpha(S)v(T)}{\alpha(S)} = v(T)$, where the last inequality is due to average monotonicity.
For the if part suppose that (1)–(3) are satisfied. In view of Remark 3.1 (3), it remains to show that, if $\alpha_S \neq 0$ and $S, T \in \mathcal{N}$ satisfy $S \subseteq T$, then $\alpha(T)v(S) \leq \alpha(S)v(T)$. To this end let $q = \frac{v(S)}{\alpha(S)}$. By (3), $\alpha(T)q \leq v(T)$, hence $\alpha(T)v(S) = \alpha(T)\alpha(S)q \leq \alpha(S)v(T)$. □

We now expand the definition of average monotonicity to NTU games in a natural way and show that average monotonic NTU games still have a nonempty core and a population monotonic allocation scheme. Indeed, the foregoing observations motivate the following definition.

Definition 3.3 The NTU game (N, V) is average monotonic if there exists $\alpha \in \mathbb{R}^N_+$ such that the following conditions hold:

$$S \in \mathcal{N} \Rightarrow 0_S \in V(S) \tag{4}$$

$$S \in \mathcal{N} \text{ and } \alpha_S = 0_S \Rightarrow 0_S \in \partial V(S) \tag{5}$$

$$S, T \in \mathcal{N}, q > 0, S \subseteq T, \alpha_S \neq 0_S, \alpha_S q \in V(S) \Rightarrow \alpha_T q \in V(T) \tag{6}$$

In this case we say that (N, V) is average monotonic w.r.t. α.

Note that the purpose of Remark 3.1 was to show how we can define average monotonicity without excluding the zero vector.

Remark 3.4 Note that an NTU game (N, V) is average monotonic w.r.t. 0_N if and only if $0_S \in \partial V(S)$ for all $S \in \mathcal{N}$. In fact, we allow $\alpha = 0_N$ for consistency reasons because, with this adjustment, a subgame (S, V_S) for $S \in \mathcal{N}$ of an average monotonic game (N, V) w.r.t. α is average monotonic w.r.t. α_S even if $\alpha_S = 0_S$.

Note that the foregoing definition generalizes the definition of average monotonicity for TU games. Indeed, in view of Proposition 3.2, a TU game (N, v) is average monotonic if and only if its corresponding NTU game (N, V_v) is average monotonic.

Lemma 3.5 *An average monotonic NTU game is weakly* superadditive.*

Proof Let (N, V) be an NTU game and let $\alpha \in \mathbb{R}^N_+$. Assume that (N, V) is average monotonic w.r.t. α and let $S \in \mathcal{N}$. If $\alpha_S = 0_S$, then $V(i) = -\mathbb{R}_+$ for all $i \in S$ and $0_S \in \partial V(S)$ so that $\times_{i \in S} V(i) \subseteq V(S)$. If $\alpha_S \neq 0_S$, we proceed as follows. For each $i \in S$ such that $\alpha_i \neq 0$, put $q_i^* = \frac{v_i}{\alpha_i}$. Then $V(i) = \{q_i \alpha_i \mid q_i \leq q_i^*\}$. By average monotonicity w.r.t. α, $q_i \alpha_S \in V(S)$ for all $q_i \leq q_i^*$. Let $q^* = \max\{q_i^* \mid i \in S, \alpha_i > 0\}$. Then $q\alpha_S \in V(S)$ for all $q \leq q^*$. As $v_j = 0$ for all $j \in S$ with $\alpha_j = 0$, $\times_{i \in S} V(i) \subseteq V(S)$. □

As shown in the following example, in contrast with TU games, if an NTU game is average monotonic, then it needs neither be weakly monotonic nor weakly superadditive.

Example 3.6 Let (N, v) with $N = \{1, 2, 3\}$ be given by $v(S) = |S|$ for all $S \subseteq N$. Consider the NTU game (N, V) that differs from (N, V_v), i.e., the NTU game corresponding to (N, v), only in as much as, for $T = \{1, 2\}$, $V(T) = \{x \in \mathbb{R}^T \mid x_1 + 3x_2 \leq 4\}$, and $V(N) = \{x \in \mathbb{R}^N \mid x(S) \leq 3 \text{ for all } S \in \mathcal{N}\}$. Then (N, V) is average monotonic w.r.t. $\alpha = (1, 1, 1)$. Moreover, $(4, 0) \in V(T)$, but $(4, 0, 0) \notin V(N)$. Therefore, (N, V) is convex-valued, but neither monotonic nor weakly superadditive. Moreover, for all $t \in \mathbb{R}$, $(4, 0, t) \notin V(N)$ so that it is also not weakly monotonic.

For each TU game (N, v) that is average monotonic w.r.t. $\alpha \in \mathbb{R}^N_+$, we recall that Izquierdo and Rafels (2001) define the *proportional distribution* w.r.t. α, $p(v, \alpha)$, by $p(v, \alpha) = 0_N$ if $\alpha = 0_N$ and $p(v, \alpha) = \alpha \frac{v(N)}{\alpha(N)}$ if $\alpha \neq 0_N$.

We generalize the definition of the proportional distribution to NTU games. For each NTU game (N, V) that is average monotonic w.r.t. $\alpha \in \mathbb{R}^N_+$, we define $p(V, \alpha) \in \mathbb{R}^N$ as follows. If $\alpha = 0_N$, then $p(V, \alpha) = 0_N$. If $\alpha \neq 0_N$, then $q^* = \max\{q \geq 0 \mid \alpha q \in V(N)\}$ exists because $V(N) \cap \mathbb{R}^N_+ \neq \emptyset$ is compact (see Definition 2.1 (ii)). In this case put $p(V, \alpha) = \alpha q^*$.

Remark 3.7 Let (N, V) be an average monotonic NTU game w.r.t. $\alpha \in \mathbb{R}^N_+$. Then,

(i) $p(V, \alpha) \in C(N, V)$.
(ii) If the game corresponds to a TU game for some TU game (N, v), then $p(V, \alpha) = p(v, \alpha)$.

4 Population monotonic allocation schemes

The notion of population monotonic allocation scheme (PMAS) for a cooperative TU game (N, v) was introduced by Sprumont (1990), and extended, in a straightforward manner, to nontransferable utility games by Moulin (1990), who investigated the monotonic core, i.e., the set of all PMAS of the game. From its definition we can directly deduce that a PMAS $x = (x^S)_{S \in \mathcal{N}}$ selects a core allocation $x^S \in C(S, V_S)$ of the subgame (S, V_S) in such a way that the payoff to a player cannot decrease when her coalition becomes larger (Izquierdo and Rafels 2001). We now recall the formal definition of a population monotonic allocation scheme.

Definition 4.1 A collection of vectors $x = (x^S)_{S\in\mathcal{N}}$ is a population monotonic allocation scheme (PMAS) of the NTU game (N, V) if and only if it satisfies the following conditions:

(1) For all $S \in \mathcal{N}$, $x^S \in \mathbb{R}^S$ and $x^S \in \partial V(S)$,
(2) For all $S, T \in \mathcal{N}$ with $S \subseteq T$, $x^S \leq x^T_S$.

Proposition 4.2 *Let (N, V) be an NTU game and $\alpha \in \mathbb{R}^N_+$. If (N, V) is average monotonic w.r.t. α, $(p(V_S, \alpha_S))_{S\in\mathcal{N}}$ is a PMAS of (N, V).*

Proof Assume that (N, v) is average monotonic w.r.t. α and let $S \in \mathcal{N}$. By Remark 3.4, (S, V_S) is average monotonic w.r.t. α_S. By the definition of the proportional solution, $p(S, V_S) = q^*_S\alpha_S$ for a unique $q^*_S \geq 0$ that satisfies $q^*_S = 0$ in the case that $\alpha_S = 0_S$. By Remark 3.7, $q^*_S\alpha_S \in C(S, v_S)$. Now, if $T \in \mathcal{N}$ satisfies $S \subseteq T$, then $q^*_T \geq q^*_S$ because either $\alpha_T = 0_T$ and, hence, $\alpha_S = 0_S$ or $\alpha_T \neq 0_T$ and, hence, $q^*_T\alpha_T \in C(T, V_T)$. In each case the proof is finished. □

As a consequence of the last proposition and the fact that every population monotonic allocation scheme assigns to each coalition a core element of the corresponding subgame, for an average monotonic game w.r.t. α, $x^S = q^*_S\alpha$ is a core element of (S, V_S).

5 Bankruptcy games with nontransferable utility

O'Neill (1982) introduced, for each TU bankruptcy problem, its corresponding TU bankruptcy game. We recall the definitions of the NTU extension of a bankruptcy problem and its corresponding NTU game as given by Orshan et al. (2003).

An NTU *bankruptcy problem* on a set N is a pair (E, c), where $E \subseteq \mathbb{R}^N$, the *estate*, is closed, comprehensive and $E \cap \mathbb{R}^N_+$ is nonempty and bounded. The vector of *claims*, $c = (c_i)_{i\in N}$ is such that $c \in \mathbb{R}^N_+ \setminus E$.

The estate, E, is the set of all feasible utility vectors, and c_i the utility claimed by player $i \in N$.

The NTU *bankruptcy game* associated to an NTU bankruptcy problem (E, c) on N, is the NTU game $(N, V_{E,c})$ defined by

$$V_{E,c}(S) = \{x_S \in \mathbb{R}^S \mid (x_S, c_{N\setminus S}) \in E\} \cup -\mathbb{R}^S_+ \text{ for all } S \in \mathcal{N}.$$

Let (N, V) be an NTU bankruptcy game. Let $S \in \mathcal{N}$. It is straightforward to check that $V(S)$ is nonempty, closed, and comprehensive.

Finally, we show that NTU bankruptcy games are average monotonic games with respect to their claims vectors.

Proposition 5.1 *Let (E, c) be an NTU bankruptcy problem. Then the bankruptcy game $(N, V_{E,c})$ is average monotonic w.r.t. c.*

Proof Let $V = V_{E,c}$. Let $S \in \mathcal{N}$. As $-\mathbb{R}^S_+ \subseteq V(S)$, $0_S \in V(S)$, i.e., (4). If $c_S = 0_S$, then $(0_S, c_{N\setminus S}) = c \notin E$, hence $0_S \in \partial V(S)$, i.e., (5). Now, assume that $c_S \neq 0_S$.

Let $T \in \mathcal{N}$ such that $S \subseteq T$. Let $q > 0$ such that $c_S q \in V(S)$. Hence, $(c_S q, c_{N\setminus S}) \in E$. As $c \notin E$ and as E is comprehensive, $q < 1$. Again as E is comprehensive, $(c_T q, c_{N\setminus T}) \in E$. We conclude that $c_T q \in V(T)$, i.e., (6). □

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Declarations

Conflict of interest All the authors of the paper certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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