Mathematical Social Sciences 125 (2023) 61–64

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Short communication

# Remarks on solidarity in bankruptcy problems when agents merge or split

Pedro Calleja [a], Francesc Llerena [b,*], Peter Sudhölter [c]

[a] Departament de Matemàtica Econòmica, Financera i Actuarial, Universitat de Barcelona-BEAT, Av. Diagonal, 690, 08034 Barcelona, Spain
[b] Departament de Gestió d'Empreses, Universitat Rovira i Virgili-ECO-SOS, Av. de la Universitat, 1, 43204 Reus, Spain
[c] Department of Economics, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark

ARTICLE INFO

*Article history:*
Received 12 April 2023
Received in revised form 10 July 2023
Accepted 11 July 2023
Available online 14 July 2023

*Keywords:*
Non-manipulability via merging or splitting
Solidarity
Bankruptcy rule

ABSTRACT

In this note, we investigate the relationship between non-manipulability via merging (splitting) and strong non-manipulability via merging (splitting). Our analysis reveals that while these two non-manipulability axioms are generally not equivalent, they do coincide when the principle of solidarity is satisfied. This principle is fulfilled by a wide range of bankruptcy rules, including parametric rules.


## 1. Introduction

Bankruptcy problems, as described by O'Neill (1982), refer to situations in which a group of agents collectively holds claims on a finite and perfectly divisible resource, but the available amount is not sufficient to fulfill all of the agents' demands. These problems are solved by rules proposing an allocation vector that takes into consideration the specifics of the agents. An important issue in economics is the study of rules that are invariant with respect to merging or splitting operations, that is, to the strategic behavior of the agents by misrepresenting their characteristics. In the setting of bankruptcy problems, a rule is *non-manipulable via merging* if no group of agents can take advantage from consolidating claims and it is *non-manipulable via splitting* if no agent can benefit from distributing her claim among a group of agents. A rule is *non-manipulable* if it is simultaneously unaffected by these two types of misrepresentations. Non-manipulability (or *strategy-proofness*) is first considered from an axiomatic perspective by O'Neill (1982) in characterizing the proportional rule in the context of bankruptcy problems. O'Neill's result was refined in different ways by Chun (1988), de Frutos (1999), Ju et al. (2007), and Calleja and Llerena (2022), among others. The implications of non-manipulability have also been explored in other contexts, such as taxation, network problems or financial systems by Ju and Moreno-Ternero (2011), Ju (2013), and Calleja et al. (2021), respectively.

Another important principle in the axiomatic approach of rules is *solidarity*, a sort of monotonicity condition concerning how a rule is affected by variations in the set of players and in the endowment or resource to distribute. Specifically, it imposes that the arrival of new agents, regardless of whether or not it is accompanied by changes in the available amount to share, should affect all the original agents in the same direction. Solidarity is introduced by Chun (1999) under the name of *population-and-resource monotonicity* and it is equivalent to the combination of two well-established requirements: *resource monotonicity* and *consistency*. Resource monotonicity says that if the amount of resource to be distributed becomes larger, no agent should be worse off. Consistency is an invariant principle with respect to population variations and requires that when a group of agents leaves with its share, then, in the reduced problem, the rule assigns the same amount as originally to the remaining agents.

On the entire domain of bankruptcy problems, Moreno-Ternero (2006) shows that non-manipulability is equivalent to *additivity of claims* (Curiel et al., 1987), or *strong non-manipulability*, requiring that merging or splitting the agents' claims do not affect the amounts received by any other agent involved in the problem. In many situations, and due to legal or practical constraints, only mergers or spin-offs are an option, but not both operations at the same time. Hence, it is worthwhile to study whether or not this reciprocity is preserved between non-manipulability via merging (splitting) and *strong non-manipulability via merging (splitting)*. In this note, we show that, in general, these axioms are not

* Corresponding author.
*E-mail addresses:* calleja@ub.edu (P. Calleja), francesc.llerena@urv.cat (F. Llerena), psu@sam.sdu.dk (P. Sudhölter).

https://doi.org/10.1016/j.mathsocsci.2023.07.004
0165-4896/

equivalent but, under the ethical principle of solidarity, they are tantamount. This issue is particularly relevant in the context of financial systems (as first introduced by Eisenberg and Noe, 2001), where several firms may fail simultaneously. Even if each defaulting firm applies a non-manipulable via merging bankruptcy rule to settle its debts, there may still be incentives for firms to merge (their liabilities, claims, and estates or endowments) in order to increase their equity value. However, if strong non-manipulable via merging bankruptcy rules are applied, no group of firms can take an advantage by merging (for more details, see Calleja et al., 2021). So, under solidarity, non-manipulability via merging is a property transferred from the bankruptcy setting to the more general financial systems environment.

The remainder of the paper is organized as follows: Section 2 introduces the model. Section 3 contains the axioms. Section 4 provides the results and Section 5 concludes.

## 2. The model

Let $\mathbb{N} = \{1, 2, \ldots\}$ (the set of natural numbers) represent the set of all potential agents (claimants) and let $\mathcal{N}$ be the collection of all non-empty finite subsets of $\mathbb{N}$. An element $N \in \mathcal{N}$ describes a finite set of agents where $|N| = n$. For each $x \in \mathbb{R}^N$ and $T \subseteq N$, $x_T$ denotes the restriction of $x$ to $T$: $x_T = (x_i)_{i\in T} \in \mathbb{R}^T$.

A *bankruptcy problem* is a problem of adjudicating claims in which a firm defaults and its available resources are not enough to satisfy its obligations with creditors.[1] Formally, a bankruptcy problem is a triple $(N, E, c)$ where $N \in \mathcal{N}$ represents the set of creditors of the firm going bankrupt; $c \in \mathbb{R}^N_+$ is the vector of claims, being $c_i$ the claim of creditor $i \in N$; and $E \geq 0$ is the net worth or estate of the firm to satisfy its obligations. Additionally, we assume that $\sum_{i\in N} c_i \geq E$. By $\mathcal{B}$ we denote the set of all bankruptcy problems.

A *bankruptcy rule* (hereafter, a rule) is a function $\beta : \mathcal{B} \longrightarrow \bigcup_{N\in\mathcal{N}} \mathbb{R}^N$ that associates with every $(N, E, c) \in \mathcal{B}$ a unique recommendation $\beta(N, E, c) \in \mathbb{R}^N$ satisfying $\sum_{i\in N} \beta_i(N, E, c) = E$ (*budget balance* (BB)), that is, the sum of all payments should be equal to the estate, and $\beta_i(N, E, c) \leq c_i$ for all $i \in N$ (*claim boundedness* (CB)), requiring that each agent should receive at most her claim. Given a bankruptcy rule $\beta$, its *dual* $\beta^d$ is defined by setting, for all $(N, E, c) \in \mathcal{B}$ and all $i \in N$, $\beta^d_i(N, E, c) = c_i - \beta_i\left(N, \sum_{i\in N} c_i - E, c\right)$. Instances of well-known rules are the *proportional* rule ($P$), the *constrained equal awards* rule ($CEA$), and the *constrained equal losses* rule ($CEL$). The $P$ rule makes awards proportional to the claims and it is probably the most commonly used rule in practice when a firm goes bankrupt. Formally, for all $(N, E, c) \in \mathcal{B}$ and all $i \in N$, $P_i(N, E, c) = \lambda c_i$ where $\lambda \in \mathbb{R}_+$ is such that $\sum_{j\in N} \lambda c_j = E$. The $CEA$ rule rewards equally to all claimants subject to no one receiving more than her claim. Formally, for all $(N, E, c) \in \mathcal{B}$ and all $i \in N$, $CEA_i(N, E, c) = \min\{c_i, \lambda\}$ where $\lambda \in \mathbb{R}_+$ is such that $\sum_{j\in N} \min\{c_j, \lambda\} = E$. In contrast, the $CEL$ rule equalizes the losses of claimants subject to no one receiving a negative amount. That is, for all $(N, E, c) \in \mathcal{B}$ and all $i \in N$, $CEL_i(N, E, c) = \max\{c_i - \lambda, 0\}$ where $\lambda \in \mathbb{R}_+$ is such that $\sum_{j\in N} \max\{c_j - \lambda, 0\} = E$. The $CEA$ and the $CEL$ are dual rules and the $P$ rule is *self-dual*, i.e., $P = P^d$. Most of the classical rules, as $P$, $CEA$, and $CEL$, are members of the so-called *parametric rules* (Young, 1987).

## 3. Axioms

In this section, we introduce several axioms related to bankruptcy rules. A considerable amount of research in this field is dedicated to studying the strategic incentives that motivate claimants to manipulate their claims by merging or splitting them to achieve extra profits. To discern between these two forms of incentives, de Frutos (1999) introduces two distinct "immunity" axioms. A rule $\beta$ on $\mathcal{B}$ satisfies

- *non-manipulability via merging* (NMM) if for all $(N, E, c)$, $(N', E, c') \in \mathcal{B}$ with $m \in N' \subset N$ such that $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c'_j = c_j$ for all $j \in N' \setminus \{m\}$, then
$$\beta_m(N', E, c') \leq \beta_m(N, E, c) + \sum_{j\in N\setminus N'} \beta_j(N, E, c).$$
- *non-manipulability via splitting* (NMS) if for all $(N, E, c)$, $(N', E, c') \in \mathcal{B}$ with $m \in N' \subset N$ such that $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c'_j = c_j$ for all $j \in N' \setminus \{m\}$, then
$$\beta_m(N', E, c') \geq \beta_m(N, E, c) + \sum_{j\in N\setminus N'} \beta_j(N, E, c).$$

The more demanding axiom of *non-manipulability* (NM) requires NMM and NMS simultaneously.

While NMM stipulates that no group of claimants can take advantage from consolidating claims; NMS, on the contrary, guarantees that no claimant can benefit from dividing its claim into claims of a group of claimants. NM imposes that agents merging or splitting receive exactly the same as initially. Ju (2003) and de Frutos (1999) study conditions under which a rule becomes either NMM or NMS. It is well known that the $CEA$ rule satisfies NMM while the $CEL$ rule satisfies NMS. Furthermore, the $P$ rule satisfies both, and it has been characterized as the unique rule satisfying NM (or equivalently, SNM) together with the mild requirement of *non-negativity* (requiring awards to be non-negative), and without imposing CB, by de Frutos (1999).[2]

These axioms can be, indeed, reformulated taking into account the effects on the agents that do not misrepresent their claims. We might require that merger of a group of agents into a single agent or the split of an agent in a multiplicity of them, affect all agents whose claims do not change in the same direction. We will, indeed, impose that all these agents receive at least as much as initially. A rule $\beta$ on $\mathcal{B}$ satisfies

- *strong non-manipulability via merging* (SNMM) if for all $(N, E, c)$, $(N', E, c') \in \mathcal{B}$ with $m \in N' \subset N$ such that $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c'_j = c_j$ for all $j \in N' \setminus \{m\}$, then
$$\beta_j(N', E, c') \geq \beta_j(N, E, c) \text{ for all } j \in N' \setminus \{m\}.$$
- *strong non-manipulability via splitting* (SNMS) if for all $(N, E, c)$, $(N', E, c') \in \mathcal{B}$ with $m \in N' \subset N$ such that $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c'_j = c_j$ for all $j \in N' \setminus \{m\}$, then
$$\beta_j(N', E, c') \leq \beta_j(N, E, c) \text{ for all } j \in N' \setminus \{m\}.$$

Curiel et al. (1987) define *additivity of claims*, renamed as *strong non-manipulability* (SNM) by Moreno-Ternero (2006), requesting both SNMM and SNMS at the same time. While SNMM and SNMS impose that each of the agents not involved in the mergers or spin offs is not worse off, SNM enforces they are compensated exactly as initially. Clearly, under BB, these are stronger versions of NMM, NMS, and NM, respectively. Moreover, as Moreno-Ternero (2006) shows, SNM and NM are equivalent requirements.

Another important property in our analysis is *solidarity*, which demands that the arrival of new agents affects all the original

[1] Probably, the most comprehensive survey on this distributive justice problem is provided by Thomson (2019).

[2] Ju et al. (2007) employ the weaker axioms of *one-sided boundedness*, stating that payoffs should be bounded from either above or below, and *pairwise non-manipulability*, that entitles agents to merge or split exclusively by pairs. Recently, Calleja and Llerena (2022) restrict the possibility to manipulate to symmetric agents or clones and provide new axiomatizations of the proportional rule making use, additionally, of a standard axiom referring monotonicity or continuity on claims.

agents in the same direction: either all gain or all lose. Formally, a rule $\beta$ on $\mathcal{B}$ satisfies

- *solidarity* (SOL) if for all $(N, E, c), (N', E', c') \in \mathcal{B}$ such that $N' \subseteq N$, if $c' = c_{N'}$, either $\beta(N', E', c') \geq \beta_{N'}(N, E, c)$ or $\beta(N', E', c') \leq \beta_{N'}(N, E, c)$.

Chun (1999) shows that solidarity is equivalent to the standard requirements of *resource monotonicity* and *consistency*. A rule $\beta$ on $\mathcal{B}$ satisfies

- *resource monotonicity* (RM) if for all pair $(N, E, c), (N, E', c) \in \mathcal{B}$ with $E' > E$, $\beta_i(N, E', c) \geq \beta_i(N, E, c)$ for all $i \in N$;
- *consistency* (CONS) if for all $(N, E, c) \in \mathcal{B}$ and all $\emptyset \neq N' \subseteq N$, $\beta_{N'}(N, E, c) = \beta\left(N', \sum_{i\in N'} \beta_i(N, E, c), c_{N'}\right)$.

The former says that no one should be worse off when the firm's assets increase and the later requires that in the reduced bankruptcy problem, which arises when some players leave with their share, each of the remaining players receives the same amount as in the original problem.

## 4. Results

In the following, we address the question of how significant is the difference between NMM and NMS and their strong counterparts. To investigate classes of rules for which SNMM and SNMS do not make a difference with their weak formulations, we show that the two are actually dual axioms. Specifically, we say that axiom A and axiom A* are *dual* if, whenever a rule $\beta$ satisfies A then its dual $\beta^d$ satisfies A*. If, moreover, A coincides with A*, then A is *self-dual*. As demonstrated by de Frutos (1999), NMM and NMS are dual to each other. In the following proposition, we show that SNMM and SNMS are also dual axioms.

**Proposition 1.** SNMM *and* SNMS *are dual to each other.*

**Proof.** Let $\beta$ be a rule satisfying SNMM and $(N, E, c), (N', E', c') \in \mathcal{B}$ with $E = E'$ and $m \in N' \subset N$ such that $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c'_j = c_j$ for all $j \in N' \setminus \{m\}$. Then, by SNMM of $\beta$ applied to $(N', \sum_{i\in N'} c'_i - E, c')$ and $(N, \sum_{i\in N} c_i - E, c)$ being $\sum_{i\in N'} c'_i - E = \sum_{i\in N} c_i - E \geq 0$ it holds that, for all $j \in N' \setminus \{m\}$

$$\begin{array}{lcll} \beta_j(N', \sum_{i\in N'} c'_i - E, c') & \geq & \beta_j(N, \sum_{i\in N} c_i - E, c) & \iff \\ c'_j - \beta_j(N', \sum_{i\in N'} c'_i - E, c') & \leq & c_j - \beta_j(N, \sum_{i\in N} c_i - E, c) & \iff \\ \beta^d_j(N', E, c') & \leq & \beta^d_j(N, E, c), & \end{array}$$

which means that $\beta^d$ satisfies SNMS.

Similarly, by reversing the direction of the chain of inequalities, it can be proven that if $\beta$ satisfies SNMS then its dual rule $\beta^d$ fulfills SNMM. □

Next, we show that non-manipulability via merging or splitting are not equivalent to their strong counterpart.

**Proposition 2.** *Neither* NMM *implies* SNMM, *nor* NMS *implies* SNMS.

**Proof.** Since SNMM and SNMS are dual axioms (Proposition 1), it is enough to prove that NMM does not imply SNMM. To do it, we define the rule $\beta$ by setting, for all $\delta = (N, E, c) \in \mathcal{B}$,

$$\beta(\delta) = \begin{cases} P(\delta) & \text{if} \quad c^*(\delta) \leq 10 \\ CEA(\delta) & \text{if} \quad c^*(\delta) > 10, \end{cases} \tag{1}$$

where $c^*(\delta) = \max_{i\in N}\{c_i\}$.

**Claim 1.** *$\beta$ does not satisfy* SNMM.

Consider the bankruptcy problem $\delta = (N, E, c)$ with set of players $N = \{1, 2, 3, 4\}$, estate $E = 12$, and vector of claims $c = (10, 6, 5, 3)$. Now, let $\delta' = (N', E, c')$ with $N' = \{1, 2, 4\}$ where agents 2 and 3 have merged into agent 2, and the vector of claims is $c' = (10, 11, 3)$. Since $c^*(\delta) = 10$, then $\beta(\delta) = P(\delta) = (5, 3, 2.5, 1.5)$. On the other hand, since $c^*(\delta') = 11$, $\beta(\delta') = CEA(\delta') = (4.5, 4.5, 3)$. Hence, $\beta_1(\delta') = 4.5 < 5 = \beta_1(\delta)$, and $\beta$ does not satisfy SNMM.

**Claim 2.** *$\beta$ satisfies* NMM.

Let $\delta = (N, E, c)$ and $\delta' = (N', E, c')$ be two bankruptcy problems such that $N' \subset N$ and there is $m \in N'$ with $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c'_j = c_j$, for all $j \in N' \setminus \{m\}$. Observe that $c^*(\delta') \geq c^*(\delta)$. Thus, we consider the following cases:

Case 1: $10 \geq c^*(\delta') \geq c^*(\delta)$.
Then, by the NMM of the proportional rule it follows directly the NMM of $\beta$.

Case 2: $c^*(\delta') \geq c^*(\delta) > 10$.
Then, by the NMM of the constrained equal awards rule it follows directly the NMM of $\beta$.

Case 3: $c^*(\delta') > 10 \geq c^*(\delta)$.
In this situation, $\beta(\delta') = CEA(\delta')$ and $\beta(\delta) = P(\delta)$. From $c^*(\delta') > c^*(\delta')$ and $c_i = c'_i$, for all $i \in N' \setminus \{m\}$, it follows that $c^*(\delta') = c'_m$. We now show by contradiction that

$$CEA_m(\delta') \leq P_m(\delta'). \tag{2}$$

If not, $CEA_m(\delta') = \min\{c'_m, \lambda\} > P_m(\delta') = \frac{E}{\sum_{i\in N'} c'_i} c'_m$ where $\lambda$ is such that $\sum_{i\in N'} \min\{c'_i, \lambda\} = E$. In case $\min\{c'_m, \lambda\} = c'_m$, due to $c'_m$ is the highest claim in $\delta'$, clearly, for all $i \in N'$ we have $CEA_i(\delta') = c'_i$ and, consequently, $E = \sum_{i\in N'} c'_i$ which implies $P_m(\delta') = c'_m$. Hence, $\min\{c'_m, \lambda\} = \lambda$ and

$$\begin{array}{rcl} \sum_{i\in N'\setminus\{m\}} CEA_i(\delta') & = & \sum_{i\in N'\setminus\{m\}} \min\{c'_i, \lambda\} \\ & \geq & \sum_{i\in N'\setminus\{m\}} \min\left\{c'_i, \frac{E}{\sum_{i\in N'} c'_i} c'_m\right\} \\ & \geq & \sum_{i\in N'\setminus\{m\}} \min\left\{c'_i, \frac{E}{\sum_{i\in N'} c'_i} c'_i\right\} \\ & = & \sum_{i\in N'\setminus\{m\}} \frac{E}{\sum_{i\in N'} c'_i} c'_i \\ & = & \sum_{i\in N'\setminus\{m\}} P_i(\delta'). \end{array}$$

But then, by BB, $E = \sum_{i\in N'} CEA_i(\delta') > \sum_{i\in N'} P_i(\delta') = E$ getting a contradiction.
Hence,

$$\begin{array}{rcl} \beta_m(\delta') & = & CEA_m(\delta') \\ & \underset{(2)}{\leq} & P_m(\delta') \\ & = & P_m(\delta) + \sum_{i\in N\setminus N'} P_i(\delta) \\ & = & \beta_m(\delta) + \sum_{i\in N\setminus N'} \beta_i(\delta), \end{array}$$

where the last but one equality comes from the NM of the proportional rule. This shows that $\beta$ satisfies NMM. □

Note that the bankruptcy rule employed in the proof of Proposition 2 (defined in (1)) inherits resource monotonicity from the resource monotonicity of both the proportional and the constrained equal awards rules. However, it can be easily shown that it fails to satisfy consistency. In the following, we prove that solidarity is crucial in order to establish the equivalence of NMM and NMS with their strong counterparts.

**Theorem 1.** *Let $\beta$ be a bankruptcy rule satisfying* SOL*. Then, $\beta$ satisfies* NMM (NMS) *if and only if it satisfies* SNMM (SNMS)*.*

**Proof.** Since NMM (SNMM) and NMS (SNMS) are dual axioms to each other (Proposition 1), it is enough to see that, under SOL, a bankruptcy rule $\beta$ satisfies NMM if and only if it satisfies SNMM. Clearly, under BB, SNMM implies NMM. To show the reverse implication, consider a bankruptcy rule $\beta$ satisfying SOL and NMM. Let $(N, E, c), (N', E', c') \in \mathcal{B}$ with $E = E'$ and $m \in N' \subset N$ such that $c'_m = c_m + \sum_{j\in N\setminus N'} c_j$ and $c_j = c'_j$ for all $j \in N' \setminus \{m\}$. By NMM, $\beta_m(N', E, c') \leq \beta_m(N, E, c) + \sum_{j\in N\setminus N'} \beta_j(N, E, c)$ or, equivalently,

$$E - \beta_m(N', E, c') \geq E - \sum_{j\in\{m\}\cup N\setminus N'} \beta_j(N, E, c). \tag{3}$$

From (3), and taking into account that $c_i = c'_i$ for all $i \in N' \setminus \{m\}$, by SOL, which implies RM and CONS, we obtain

$$\begin{aligned}
\beta_i(N', E, c') &\underset{\text{CONS}}{=} \beta_i\left(N' \setminus \{m\}, \sum_{i\in N'\setminus\{m\}} \beta_i(N', E, c'), c'_{N'\setminus\{m\}}\right)\\
&\underset{\text{BB}}{=} \beta_i\left(N' \setminus \{m\}, E - \beta_m(N', E, c'), c'_{N'\setminus\{m\}}\right)\\
&\underset{\text{RM}}{\geq} \beta_i\left(N' \setminus \{m\}, E - \sum_{j\in\{m\}\cup N\setminus N'} \beta_j(N, E, c), c_{N'\setminus\{m\}}\right)\\
&\underset{\text{BB}}{=} \beta_i\left(N' \setminus \{m\}, \sum_{i\in N'\setminus\{m\}} \beta_i(N, E, c), c_{N'\setminus\{m\}}\right)\\
&\underset{\text{CONS}}{=} \beta_i(N, E, c),
\end{aligned}$$

which proves SNMM of $\beta$.

Note that BB and CB ensure that the reduced problems are well defined. □

Observe that the equivalence stated in Theorem 1 does not impose a requirement for the rules to be non-negative, allowing for the possibility of negative payoffs.

The well-established class of *parametric rules* introduced by Young (1987) satisfies SOL. Indeed, *parametric rules* are characterized by means of consistency, together with *continuity* and *symmetry*. While continuity guarantees that small changes on both the claims and the endowment do not provoke large changes on the rule, symmetry imposes that agents with the same claim are rewarded equally (for formal definitions, see Thomson, 2019). These rules also satisfy RM, thereby ensuring SOL. Hence, a direct consequence of Theorem 1 is the following.

**Corollary 1.** *A parametric rule is* NMM (NMS) *if and only if it is* SNMM (SNMS).

Thus, the *CEA* and the *CEL* rules satisfy SNMM and SNMS, respectively, while the *P* rule satisfies both axioms. Other parametric rules satisfying SNMM are the canonical constrained egalitarian and Piniles' rules (for definitions of these rules, see Thomson, 2019 ).[3] Since the dual of a parametric rule is also parametric and SNMM and SNMS are dual properties, the dual of the Piniles' rule and the dual of the constrained egalitarian rule comply with SNMS.

## 5. Final comments

In the setting of bankruptcy problems, we have shown that non-manipulability via merging or splitting is not equivalent to their strong counterpart. However, we have found that the principle of solidarity can bridge this gap. This implies that for a wide range of rules, including parametric rules (Young, 1987), both axioms are coincident. Solidarity is equivalent to the combination of resource monotonicity and consistency (Chun, 1999) and, as we have observed, the rule introduced in the proof of Proposition 2 satisfies the former, but not the latter. Therefore, to test if Theorem 1 is tight, it remains open to investigate if there are rules satisfying non-manipulability via merging (splitting) and consistency but neither strong non-manipulability via merging (splitting) nor resource monotonicity. Although our intuition is that it is, this is a challenging problem since most classical bankruptcy rules exhibit resource monotonicity. Moreover, in the presence of consistency (in fact, bilateral consistency is enough), symmetry and resource continuity imply resource monotonicity. Therefore, any rule that deviates from resource monotonicity must either sacrifice symmetry or resource continuity, making it seem ad-hoc or arbitrarily constructed. Finally, under consistency (bilateral being sufficient) resource monotonicity for two-agent problems implies resource monotonicity for any set of claimants and, thus, these rules must fail resource monotonicity for two-agents problems, too.[4]

## Data availability

No data was used for the research described in the article.

## Acknowledgments

The first two authors acknowledge support from research grants PID2020-113110GB-I00 (MINECO 2020, Spain) and 2021S GR00306 (AGAUR, Spain, Generalitat de Catalunya). The first author also acknowledges support from Universitat de Barcelona, Spain under research grant AS017672. The second author also acknowledges support from research grants PID2019-105982GB-I00 (MINECO 2019, Spain) and 2019PFR-URV-B2-53. The authors are grateful to an associate editor of this journal for his comments that helped to improve the writing of this paper.

## References

Calleja, P., Llerena, F., 2022. Non-manipulability by clones in bankruptcy problems. Econom. Lett. 221, 110921.
Calleja, P., Llerena, F., Sudhölter, P., 2021. On manipulability in financial systems. Discussion Papers on Economics No. 8/2021, University of Southern Denmark (SDU).
Chun, Y., 1988. The proportional solution for rights problems. Math. Social Sci. 15, 231–246.
Chun, Y., 1999. Equivalence of axioms for Bankruptcy problems. Internat. J. Game Theory 28, 511–520.
Curiel, I.J., Maschler, M., Tijs, S.H., 1987. Bankruptcy games. Zeitschrift für Oper. Res. 31, A143–A159.
de Frutos, M.A., 1999. Coalitional manipulations in a Bankruptcy problem. Rev. Econ. Des. 4, 255–272.
Eisenberg, L., Noe, T.H., 2001. Systemic risk in financial systems. Manage. Sci. 47 (2), 236–249.
Ju, B.-G., 2003. Manipulation via merging and splitting in claims problems. Rev. Econ. Des. 8, 205–215.
Ju, B.-G., 2013. Coalitional manipulation on networks. J. Econom. Theory 148, 627–662.
Ju, B.-G., Miyagawa, E., Sakai, T., 2007. Non-manipulable division rules in claims problems and generalizations. J. Econom. Theory 132, 1–26.
Ju, B.-G., Moreno-Ternero, J.D., 2011. Progressive and merging-proof taxation. Internat. J. Game Theory 40, 43–62.
Moreno-Ternero, J.D., 2006. Proportionality and non-manipulability in Bankruptcy problems. Int. Game Theory Rev. 1, 127–139.
O'Neill, B., 1982. A problem of rights arbitration from the Talmud. Math. Social Sci. 2, 345–371.
Thomson, W., 2019. How to Divide when There Isn'T Enough: From Aristotle, the Talmud, and Maimonides to the Axiomatics of Resource Allocation. In: Econometric Society Monograph, Cambridge University Press.
Young, H.P., 1987. On dividing an amount according to individual claims or liabilities. Math. Oper. Res. 12 (3), 398–414.

[3] To prove that these parametric rules satisfy the axioms we refer readers to Proposition 1 in Ju (2003) which characterizes the set of parametric rules that are NMM (NMS).

[4] For detailed explanations of these implications see Thomson (2019).